The theoretical expressions of wood thermal diffusivity

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Abstract: From viewpoint of chemical element and microstructure of wood, this paper makes a discussion on thermal diffusivity of wood and two theoretical expressions of thermal diffusivity for the choral and radial directions were derived. The thermal diffusivities of the choral and radial directions for about 20 species of trees were calculated with the derived theoretical expressions and compared with the experimental values. The average error of the theoretical values of thermal diffusivity was 7.5% for choral direction and 6.2% for radial direction.

Key words: Thermal diffusivity; Theoretical expression; Wood
CLC number: S781.3 **Document code:** A **Article ID:** 1007-662X(2001)02-0133-03

Theoretical derivation of wood thermal diffusivity

The chemical element and structure of wood are so complicated that, due to the difficulty in the theoretical studying of the thermal diffusivity, there has not been any research achievement reported on the theoretical study of the wood thermal diffusivity so far.

The thermal diffusivity indicates the ability which enables the temperature to be the same in the body, when the body in the heating or cooling process. The greater the thermal diffusivity of the body, the less the temperature difference inside the body. It follows that thermal diffusivity of the material is directly proportional to thermal conductivity and inversely proportional to the heat capacity of the unit volume material. It is defined as

$$a = \frac{\lambda}{c\rho} \tag{1}$$

where λ is the body thermal conductivity, c is the body specific heat, ρ is the body mass density.

It appears from equation (1) that the theoretical expressions of thermal conductivity and specific heat of wood are required to set up theoretical expression of the wood thermal diffusivity.

Theoretical expression of wood specific heat

The theoretical expression of specific heat of dry wood was derived by thermodynamics from chemical composition and microstructure of wood (Yang 1988, 1993, 1998).

$$C_o(T) = \frac{R}{\mu}[6D(\frac{\Theta_D}{T}) + \sum_{v=1}^{57} E(\frac{\Theta_v}{T})] \tag{2}$$

where $D(\frac{\Theta_D}{T})$ is Debye function, $E(\frac{\Theta_v}{T})$ is Einstein function.

Because the computing work is too heavy by equation (2), we must simplify treatment of equation (2). Therefore, we can expand equation (2) into Taylor series at $T=T_0$ (=273.15K) (Yang 1991), then, substituting known quantity, one obtains

$$C_0(t)=1.2294+0.006714t \quad (\mathrm{kJ \cdot kg^{-1} \cdot K^{-1}}) \tag{3}$$

by equation (3), the specific heat of the wet wood C_w (whose moisture content is w%) is easily derived

$$C_w(t, w) = \frac{1.2294 + 0.006714\,t + 0.04187\,w}{1 + 0.01w} \quad (\mathrm{kJ \cdot kg^{-1} \cdot K^{-1}}) \tag{4}$$

Theoretical expressions of thermal conductivity of wood

In order to derive the theoretical expression of thermal conductivity of wood, we put forward that the phonon is the carrier of energy in heat conduction of wood (Yang 1997). We can derive the theoretical expression of thermal conductivity of dry wood, and obtain (Yang 1987, 1993,1992)

$$\lambda_o = \frac{R}{M}\rho\, uLD\ (\frac{\Theta_D}{T}) \tag{5}$$

where M is molar mass of wood cell, ρ is mass density of dry wood, L is mean free path of the phonon, and other signs mean the same as stated above.

We assume that functional relation between the mean free path of the phonon and the temperature, Debye fre-

Foundation item: The project was supported by Natural Science Foundation of Fujian Province. "Theoretical Research on Wood Thermal Property".
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Received date: 2000-08-26
Responsible editor: Song Funan

quency and the density of wood is as follows (Yang 1999).

$$L = C_1\left(\frac{T}{\Theta_D}\right)^{3/2} \exp\left(C_2\sqrt{\frac{\rho_A}{\rho}}\right) \quad (6)$$

Where ρ_A is the air density, its numerical value may take the value at normal state. C_1 and C_2 are two constants awaiting determination by means of experimental value of wood thermal conductivity in the choral and radial direction respectively. It accuracy is assured, we may take $C_1=10.05\times10^{-10}$ m and $C_2=8$ for the heat current of the choral direction and $C_1=12.60\times10^{-10}$ m and $C_2=5.283$ for the heat current of the radial direction.

In order to simplify calculation, we can expand Debye function in equation (5) into Taylor series at T=273.15 K, and expand $T^{3/2}$ in equation (6) into power series of t. Then, by substituting known numerical value, we can obtain respectively simplified formula of wood thermal conductivity in the choral and radial directions.

$$\lambda_{cho.} = (9.166+0.05139t)\times10^{-3}\frac{\rho}{\sqrt{u}}\exp\left(\frac{9.086}{\sqrt{\rho}}\right) \quad (7)$$

$(\mathrm{w}\cdot\mathrm{m}^{-1}\cdot\mathrm{K}^{-1})$

$$\lambda_{rad.} = (11.49+0.0645t)\times10^{-3}\frac{\rho}{\sqrt{u}}\exp\left(\frac{6}{\sqrt{\rho}}\right) \quad (8)$$

$(\mathrm{w}\cdot\mathrm{m}^{-1}\cdot\mathrm{K}^{-1})$

The relationship between thermal conductivity of wet wood (whose moisture content is w%) and that of dry wood is shown as the following equation (Kollmann 1968)

$$\lambda_w=(1+1.25w\%)\lambda \quad (9)$$

The theoretical expression of wood thermal diffusivity

Substituting equations (4) (7) (9) and equations (4) (8) (9) respectively into equation (1), we can obtain respectively theoretical expressions of wood thermal diffusivity in the choral and radial directions

$$a_{cho.} = \frac{(9.166+0.05139t)(1+0.0225w)\exp(9.086/\sqrt{\rho})\times10^{-6}}{\sqrt{u}(1.2294+0.006714t+0.04187w)} \quad (10)$$

$(\mathrm{m}^2\cdot\mathrm{s}^{-1})$

$$a_{rad.} = \frac{(11.49+0.0645t)(1+0.0225w)\exp(6/\sqrt{\rho})\times10^{-6}}{\sqrt{u}(1.2294+0.006714t+0.04187w)} \quad (11)$$

$(\mathrm{m}^2\cdot\mathrm{s}^{-1})$

Comparison between theoretical values and experimental values

By the equation (10), we calculated the thermal diffusivity in the choral direction of 22 species of trees, and compared it with experimental values, as shown in Table 1.

Table 1. Theoretical and experimental values of the thermal diffusivity in choral direction

Species of trees	Moisture content W /%	Temperature. T /℃	Density ρ /kg · m⁻³	Sound velocity U /m · s⁻¹	Thermal diffusivity /10⁻⁶m² · s⁻¹		Error /%
					Theoretical	Experimental	
Pinus Koraiensis	11.0	16.5	456	5057	0.1493	0.1344	11.1
Cunninghamia lanceolata	17.0	23.5	459	5079	0.1465	0.1308	12.0
Pinus massoniana	14.6	24.2	490	4958	0.1478	0.1406	5.1
Podocarpus imbricatus	13.6	17.4	529	4248	0.1562	0.1683	7.2
Larix olgensis	14.5	13.9	702	4658	0.1402	0.1306	7.3
Paulownia fargesii	10.5	17.2	253	4394	0.1859	0.1717	8.3
P. fortunei	13.0	21.7	246	4386	0.1866	0.1753	6.4
P. catalpifolia	12.7	22.4	312	4442	0.1742	0.1558	11.8
P. tomentosa var. Inlingensis	11.3	17.6	321	4360	0.1745	0.1697	2.8
P tomentosa	13.0	22.4	341	4489	0.1692	0.1583	6.9
Alniphyllum fortunei	13.9	24.5	450	4609	0.1566	0.1572	0.4
Tilia mandshurica	10.9	16.0	470	5008	0.1490	0.1306	14.1
Juglans mandshurica	14.5	16.1	481	4973	0.1462	0.1342	9.0
Melia azedarach	15.3	24.5	486	4532	0.1545	0.1517	1.8
Betula platyphylla	13.2	16.7	583	4957	0.1420	0.1283	10.7
Acer davidii	12.3	24.5	616	4310	0.1532	0.1472	4.1
Acer mono	11.7	25.0	659	4471	0.1490	0.1469	1.5
B. alnoides	15.3	24.2	674	4879	0.1399	0.1524	1.8
Fraxinus mandshurica	12.1	26.0	702	4882	0.1410	0.1247	13.1
Quercus mongolica	12.7	25.0	721	4288	0.1492	0.1342	11.2
Homalium hainanense	12.0	24.0	900	4494	0.1429	0.1239	13.7
Quercus acutissima	14.8	21.0	963	4433	0.1381	0.1328	4.0

It is clear from Table 1 that the maximum deviation from the experimental value is 14.1%, and the average error is 7.5%.

By the equation (11), we calculated the thermal diffusivity in the radial direction of 18 species of trees, and compared it with experimental values, as shown in Table 2.

Table 2. Theoretical and experimental values of the thermal diffusivity in radial direction

Species of trees	Moisture content w /%	Temp. t ℃	Density ρ kg • m^{-3}	Sound velocity u /m • s^{-1}	Thermal diffusivity /10^{-6}m^2 • s^{-1}		Error /%
					Theoretical	Experimental	
Pinus Koraiensis	11.7	16.5	438	5057	0.1623	0.1536	5.6
Pinus massoniana	15.0	24.2	531	4958	0.1592	0.1386	14.8
Paulownia fargesii	10.2	20.6	250	4394	0.1936	0.1897	2.0
P. fortunei	13.3	21.7	262	4386	0.1890	0.1989	5.0
P. catalpifolia	12.7	22.4	327	4442	0.1819	0.1703	6.5
P. tomentosa var. tsinlingensis	10.6	17.6	306	4360	0.1863	0.1711	8.9
P tomentosa	12.7	22.4	335	4489	0.1801	0.1661	8.4
Alniphyllnus fortunei	14.1	24.5	418	4609	0.1714	0.1817	5.7
Tilia mandshurica	10.6	16.0	416	5008	0.1651	0.1519	8.7
Juglans mandshurica	13.4	16.1	481	4973	0.1600	0.1539	4.3
Catalpa duolouxii	12.6	26.0	472	4688	0.1684	0.1494	12.7
Melia azedarach	15.1	24.5	420	4532	0.1720	0.1625	5.9
Betula platyphylla	12.6	16.7	596	4957	0.1567	0.1480	5.9
Acer davidii	11.5	24.5	589	4310	0.1711	0.1764	3.0
Acer mono	12.0	25.0	670	4471	0.1652	0.1728	4.4
B. alnoides	15.0	24.2	752	4879	0.1540	0.1514	1.7
Fraxinus mandshurica	12.1	26.0	680	4882	0.1579	0.1436	10.0
Quercus mongolica	11.5	25.0	637	4288	0.1700	0.1589	7.2

It is evident from Table 2 that the maximum deviation from the experimental value is 14.8%, and average error is 6.2%.

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